Principles of Communications

EES 351

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4.6 Bandwidth-Efficient Modulations

EES 351: Principles of Communications **2021/1**

HW 8 — Due: October 27, 11:59 PM

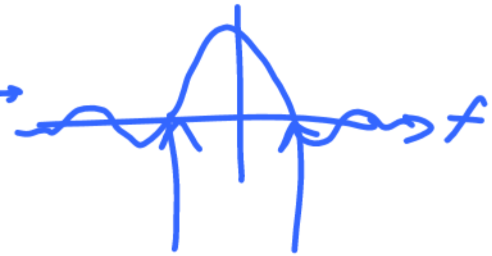
Lecturer: Prapun Suksompong, Ph.D.

Problem 3. Consider a signal $g(t)$. Recall that $|G(f)|^2$ is called the **energy spectral density** of $g(t)$. Integrating the energy spectral density over all frequency gives the signal's total energy. Furthermore, the energy contained in the frequency band I can be found from the integral $\int_I |G(f)|^2 df$ where the integration is over the frequencies in band I. In particular, if the band is simply an interval of frequency from f_1 to f_2, then the energy contained in this band is given by

$$\int_{f_1}^{f_2} |G(f)|^2 df. \tag{7.1}$$

In this problem, assume

$$g(t) = 1[-1 \le t \le 1].$$



(a) Find the (total) energy of $g(t)$.

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} (1[-1 \le t \le 1])^2 dt = \int_{-1}^{1} 1\, dt = 2.$$

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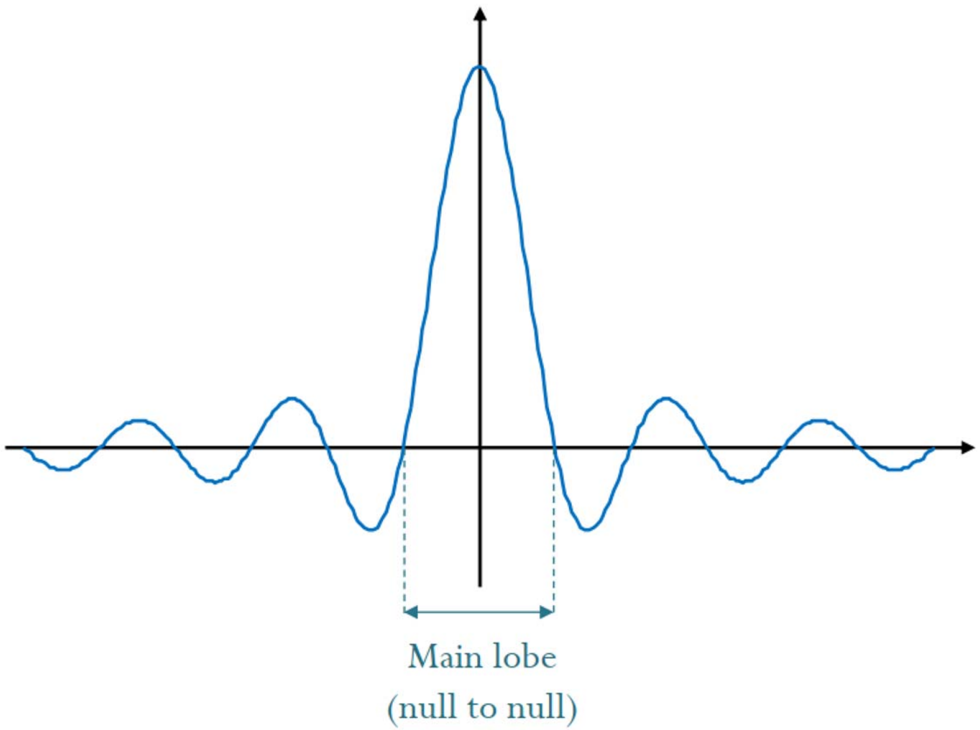
(b) Figure 6.2 define the main lobe of a sinc pulse. It is well-known that the main lobe of the sinc function contains about 90% of its total energy. Check this fact by first computing the energy contained in the frequency band occupied by the main lobe and then compare with your answer from part (a).



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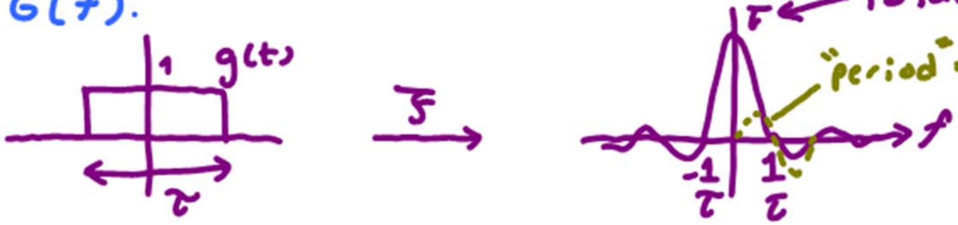
First, we need $G(f)$.

Recall that



τ ← total area in time-domain

"period" $= \frac{2}{\tau}$ → "freq" $= \frac{\tau}{2}$ → $\sin\left(2\pi \frac{\tau}{2} f\right)$

$G(f) = \tau \operatorname{sinc}(\pi f \tau)$

Here, $\tau = 2$. So, $G(f) = 2\operatorname{sinc}(2\pi f)$

The main lobe occupies an interval of frequency from $f_1 = -\frac{1}{\tau} = -\frac{1}{2}$ to $f_2 = +\frac{1}{\tau} = +\frac{1}{2}$.

So, the energy contained in the band $B = [f_1, f_2]$ is given by $\int_{-1/2}^{1/2} \left(2\operatorname{sinc}(2\pi f)\right)^2 df \approx 1.8056$ ← MATLAB

Compared with the answer from part (a), this is $\approx 90\%$ of the total energy.

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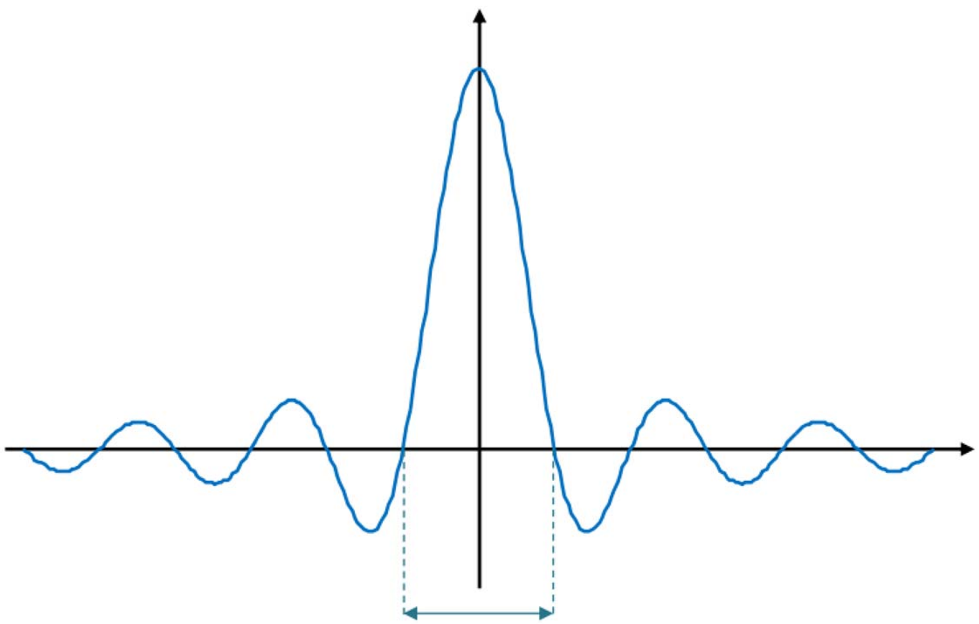
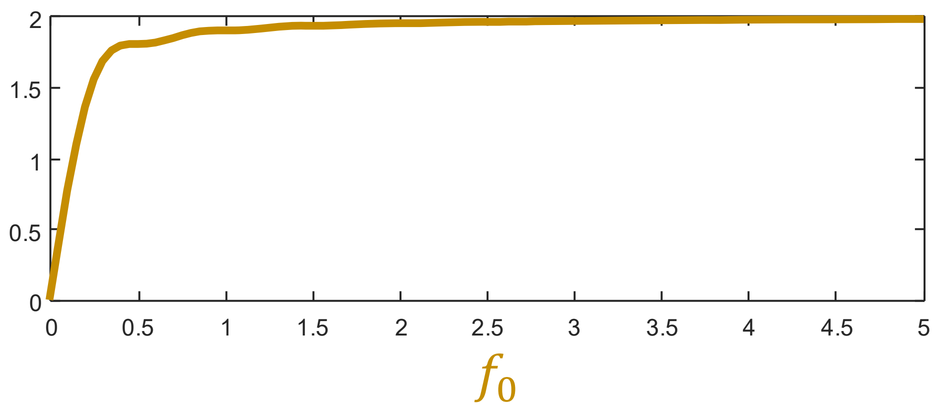
(c) Suppose we want to include more energy by considering wider frequency band. Let this band be the interval $I = [-f_0, f_0]$. Find the minimum value of f_0 that allows the band to capture at least $\boxed{99\%}$ of the total energy in $g(t)$.

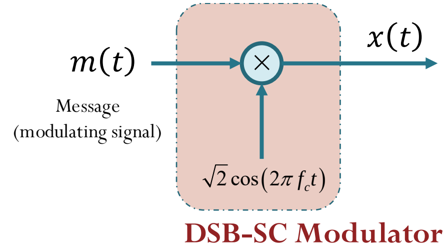
Using MATLAB, we can look at the fraction of energy as a function of f_0. We found that at around $f_0 \approx 5.1$, the fraction begins to exceed 99%.

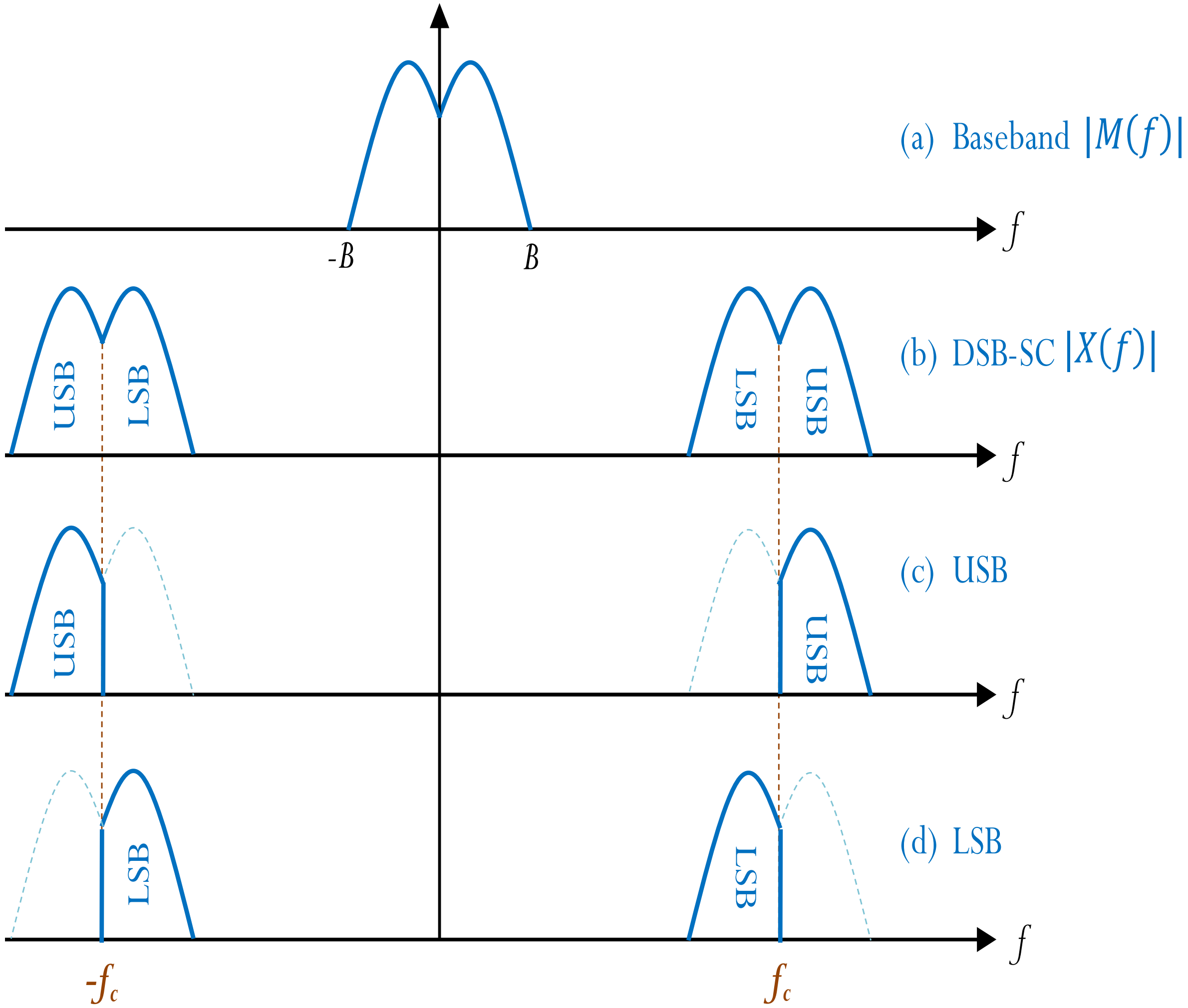


$$\int_{-f_0}^{f_0} \left(2\operatorname{sinc}(2\pi f)\right)^2 df$$



DSB = double sidebands





[Figure 31 in Example 4.84]

- [2.30] When $m(t)$ is real-valued, its spectrum $M(f)$ has **conjugate symmetry**.
- [4.9] With such message, the corresponding modulated signal's spectrum $X(f)$ will also inherit the symmetry but now centered at f_c (instead of at 0).
- The portion that lies above f_c is known as the **upper sideband (USB)** and the portion that lies below f_c is known as the **lower sideband (LSB)**.
- Similarly, the spectrum centered at $-f_c$ has upper and lower sidebands.